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3. $b : b' :: 4 : 3$. 3rd condition.
4. $v : v' :: 21 : 21$, multiplying and reducing, and remembering that the value $\propto l.b.q$.
5. Also $v - v' = \$500$. Whence,
6. $v = \$10500$. From (4) and (5),
7. $v' = \$10000$.

This problem was also solved by *B. F. SINE, NELSON S. RORAY, P. S. BERG, F. M. McGAW, J. C. CORBIN, COOPER D. SCHMITT, FREDERIC R. HONEY, H. C. WILKES*, and *G. B. M. ZERR*. M. A. Gruber sent in a solution of Problem 70, Department of Arithmetic, too late for credit in last issue. His answer is 6.48 years.



ALGEBRA.

Conducted by J. M. COLAW, Monterey, Va. All contributions to this department should be sent to him.

SOLUTIONS OF PROBLEMS.

68. Proposed by ROBERT JUDSON ALEY, M. A., Professor of Mathematics in Indiana University, Bloomington, Indiana.

Sum to n terms the series, $n\cos\theta + (n-1)\cos 2\theta + (n-2)\cos 3\theta$, etc.

[*Chrystal's Algebra.*]

I. Solution by O. W. ANTHONY, M. Sc., Professor of Mathematics in Columbian University, Washington, D. C.

Let $S = n\cos\theta + (n-1)\cos 2\theta + (n-2)\cos 3\theta + \dots$. Also let $S_s = \sin\theta + \sin 2\theta + \sin 3\theta + \dots$, and $S_c = \cos\theta + \cos 2\theta + \cos 3\theta + \dots$.

$$\begin{aligned} S &= n[\cos\theta + \cos 2\theta + \cos 3\theta + \dots] - [\cos 2\theta + 2\cos 3\theta + \dots], \\ &= (n+1)[\cos\theta + \cos 2\theta + \cos 3\theta + \dots] - [\cos\theta + 2\cos 2\theta + 3\cos 3\theta + \dots], \\ &= (n+1)S_c - dS_s/d\theta. \end{aligned}$$

Now $S_s = [\cos\{\frac{1}{2}(n+1)\theta\} \sin\{\frac{1}{2}(n\theta)\}] / \sin\frac{1}{2}\theta$, and $S_c = [\sin\{\frac{1}{2}(n+1)\theta\} \sin\{\frac{1}{2}(n\theta)\}] / \sin\frac{1}{2}\theta$.

$$\therefore S = (n+1) \frac{\cos\{\frac{1}{2}(n+1)\theta\} \sin\{\frac{1}{2}(n\theta)\}}{\sin\frac{1}{2}\theta} - \frac{d}{d\theta} \left[\frac{\sin\{\frac{1}{2}(n+1)\theta\} \sin\{\frac{1}{2}(n\theta)\}}{\sin\frac{1}{2}\theta} \right],$$

probably as compact a form as can be obtained.

II. Solution by G. B. M. ZERR, A. M., Ph. D., Texarkana, Arkansas.

Let $S = \text{sum required}$,

$$2\sin\frac{1}{2}\theta \cos n\theta = \sin\{\theta + \frac{2n-1}{2}\theta\} - \sin\{\theta + \frac{2n-3}{2}\theta\}$$

$$4\sin\frac{1}{2}\theta\cos(n-1)\theta = 2\sin\left\{\theta + \frac{2n-3}{2}\theta\right\} - 2\sin\left\{\theta + \frac{2n-5}{2}\theta\right\}$$

$$6\sin\frac{1}{2}\theta\cos(n-2)\theta = 3\sin\left\{\theta + \frac{2n-5}{2}\theta\right\} - 3\sin\left\{\theta + \frac{2n-7}{2}\theta\right\}$$

$$8\sin\frac{1}{2}\theta\cos(n-3)\theta = 4\sin\left\{\theta + \frac{2n-7}{2}\theta\right\} - 4\sin\left\{\theta + \frac{2n-9}{2}\theta\right\}$$

.....

$$2n\sin\frac{1}{2}\theta\cos\theta = n\sin(\theta + \frac{1}{2}\theta) - n\sin(\theta - \frac{1}{2}\theta).$$

Adding we get

$$2S\sin\frac{1}{2}\theta = (\sin\frac{3}{2}\theta + \sin\frac{5}{2}\theta + \sin\frac{7}{2}\theta + \dots + \sin\frac{2n+1}{2}\theta) - n\sin\frac{1}{2}\theta,$$

$$= [\sin(\frac{n+2}{2})\sin\frac{1}{2}(n\theta)]/\sin\frac{1}{2}\theta - n\sin\frac{1}{2}\theta.$$

$$\therefore S = [\sin(\frac{n+2}{2}\theta)\sin\frac{1}{2}(n\theta) - n\sin^2\frac{1}{2}\theta]/2\sin^2\frac{1}{2}\theta.$$

The series in parenthesis above is summed in all trigonometries in the series, $\sin\alpha + \sin(\alpha + \beta) +$, etc., by making $\alpha = \frac{3}{2}\theta$, $\beta = \theta$.

III. Solution by J. SCHEFFER, A. M., Hagerstown, Maryland.

The given series may be broken up into :

$$n[\cos\theta + \cos 2\theta + \cos 3\theta + \dots + \cos n\theta]$$

$$- [\cos 2\theta + 2\cos 3\theta + 3\cos 4\theta + \dots + (n-1)\cos n\theta].$$

To sum the series $\cos\theta + \cos 2\theta + \cos 3\theta + \dots + \cos n\theta$, we have

$$\sin\frac{1}{2}\theta - \sin\frac{3}{2}\theta = -2\cos\theta\sin\frac{1}{2}\theta.$$

$$\sin\frac{3}{2}\theta - \sin\frac{5}{2}\theta = -2\cos 2\theta\sin\frac{1}{2}\theta.$$

$$\sin\frac{5}{2}\theta - \sin\frac{7}{2}\theta = -2\cos 3\theta\sin\frac{1}{2}\theta.$$

.....

$$\sin\frac{1}{2}(2n-1)\theta - \sin\frac{1}{2}(2n+1)\theta = -2\cos n\theta\sin\frac{1}{2}\theta.$$

Adding, we have, $\sin\frac{1}{2}\theta - \sin\frac{1}{2}(2n+1)\theta = -2\sin\frac{1}{2}\theta \Sigma(n\theta)$.

$$\therefore \Sigma(n\theta) = [\sin\frac{1}{2}(2n+1)\theta - \sin\frac{1}{2}\theta]/2\sin\frac{1}{2}\theta = [\cos\frac{1}{2}(n+1)\theta\sin\frac{1}{2}n\theta]/\sin\frac{1}{2}\theta.$$

$$\therefore n\Sigma(n\theta) = [n\cos\frac{1}{2}(n+1)\theta\sin\frac{1}{2}n\theta]/\sin\frac{1}{2}\theta.$$

To sum the second part, we have,

$$x^2 + 2x^3 + 3x^4 + \dots + (n-1)x^n = [x^2 - nx^{n+1} + (n-1)x^{n+2}]/(1-x)^2.$$

Putting $x = \cos\theta + i\sin\theta$, and employing the formula $(\cos\theta + i\sin\theta)^m = \cos mb + i\sin mb$, we obtain after putting the real parts of both members equal, and making all necessary reductions, for the sum of the second series

$$= \frac{\cos\theta - n\cos n\theta + (n-1)\cos(n+1)\theta}{4\sin^2 \frac{1}{2}\theta};$$

so that the sum of the given series

$$= \frac{n \cos \frac{1}{2}(n+1)\theta \sin \frac{1}{2}n\theta}{\sin \frac{1}{2}\theta} + \frac{\cos \theta - n \cos n\theta + (n-1) \cos(n+1)\theta}{4 \sin^2 \frac{1}{2}\theta}.$$

To test this formula we must of course, leave the coefficient n of the first expression unchanged, while in all the other factors and terms which involve n , n must be put successively = 1, 2, 3, 4, etc.

Also solved by *E. W. MORRELL.*

69. Proposed by C. E. WHITE, A. M., Trafalgar, Indiana.

Prove that $x^n \pm x^{n-1} + x^{n-2} \pm \dots + (\pm 1)^{n-1}x + (\pm 1)^n = (x \mp 1)^n \pm A(x \mp 1)^{n-1} + B(x \mp 1)^{n-2} \pm \dots + (\pm 1)^n x$, where A, B, C, \dots are the binomial coefficients of the $(n+1)$ th order.

I. Solution by O. W. ANTHONY, M. Sc., Professor of Mathematics in Columbian University, Washington, D. C.

$$\{[(x+1)-1]^{n+1}+1\}/(x+1), = (x-1)^n + C_{n+1}^2(x-1)^{n-1} + C_{n+1}^3(x-1)^{n-2} + \dots,$$

$$\text{or } (x+1)^n = C_{n+1}^2(x+1)^{n-1} + C_{n+1}^3(x-1)^{n-2} - \dots,$$

$$\text{or } (x+1)^n = C_{n+1}^2(x+1)^{n-1} + C_{n+1}^3(x-1)^{n-2} - \dots,$$

$$=(x\mp 1)^n\pm A(x\mp 1)^{n-1}+B(x\mp 1)^{n-2}+\dots$$

II. **Solution by E. W. MORRELL, Professor of Mathematics in Montpelier Seminary, Montpelier, Vermont.**

Let $K = x^n \pm x^{n-1} + x^{n-2} \pm \dots + (\pm 1)^{n-1}x + (\pm 1)^n$.

Put $x=y=1$, expanding and observing that the sign of the last term of each expression is \pm if n is odd but $+$ if n is even, we may write :

$$x^n = (y \pm 1)^n = y^n \pm ny^{n-1} + \frac{1}{2}[n(n-2)]y^{n-2} \pm \dots + (\pm 1)^{n-1}ny + (\pm 1)^n$$

$$\pm x^{n-1} = \pm (y \pm 1)^{n-1} = \pm y^{n-1} + (n-1)y^{n-2} \pm \dots + (\pm 1)^{n-1}(n-1)y + (\pm 1)^n$$

$$x^{n-2} = (y \pm 1)^{n-2} = \dots \dots \dots y^{n-2} \pm \dots \dots \dots + (\pm 1)^{n-1}(n-2)y + (\pm 1)^n$$

etc etc